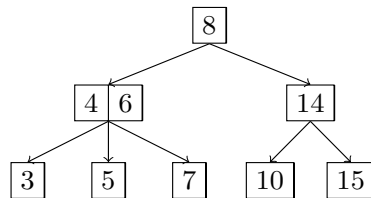


1 2-3 Trees and LLRB's

- (a) Draw what the following 2-3 tree would look like after inserting 18, 38, 12, 13, and 20.



- (b) Now, convert the resulting 2-3 tree to a left-leaning red-black tree.

- (c) If a 2-3 tree has depth H (that is, the leaves are at distance H from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

2 Hashing

- (a) Here are three potential implementations of the `Integer`'s `hashCode()` function. Categorize each as either a valid or an invalid hash function. If it is invalid, explain why. If it is valid, point out a flaw or disadvantage.

```
public int hashCode() {  
    return -1;  
}
```

```
public int hashCode() {  
    return intValue() * intValue();  
}
```

```
public int hashCode() {  
    return super.hashCode();  
}
```

- (b) For each of the following questions, answer **Always**, **Sometimes**, or **Never**.
1. When you modify a key that has been inserted into a `HashMap` will you be able to retrieve that entry again? Explain.

 2. When you modify a value that has been inserted into a `HashMap` will you be able to retrieve that entry again? Explain.

3 Even More Asymptotics *Extra*

Give the runtime of the following functions in theta notation.

(a) $\Theta(\quad)$

```

1 public static void f1(int N) {
2     for (int i = 2; i < N; i *= i) { }
3     System.out.println("Hi");
4 }
```

(b) $\Theta(\quad)$

```

1 public static void f2(int N) {
2     for (int i = 0; i < N; i++) {
3         int jLimit = Math.pow(2, i + 1) - 1;
4         for (int j = 0; j < jLimit; j += 2) {
5             System.out.println("Hi");
6         }
7     }
8 }
```

(c) *This problem is really hard and not in scope but its fun.*

$\Theta(\quad)$

```

1 public static void f3(int N) {
2     for (int i = 0; i < N * N; i++) {
3         for (int j = 0; j < i; j *= 2) {
4             System.out.println("Hi");
5         }
6     }
7 }
```