CS 61B — Homework 3

Fall 2020 — October 7, 2020

1 Asymptotic Warm Up

Give the tightest asymptotic bound on foo(n).

```
public int foo(int n) {
    if (n == 0) {
        return 0;
    }
    bloop(n);
    return foo(n / 3) + foo(n / 3) + foo(n / 3);
}

public int bloop(int n) {
    for (int i = 0; i < n; i += 1) {
        System.out.println("Ah, loops too");
    }
    return n;
}
```

2 Asymptotic Potpourri

Note: This is the hardest problem on this Homework. If you are stuck on it for a long time, move on to other problems, and post on Ed or come to Office Hours so we can help you.

For the following methods, give the runtime of in Θ notation. Your answer should be a function of N that is as simple as possible with no unnecessary leading constants or lower order terms.

(a) Give the runtime of `mystery1(n)` in Θ notation.

```java
public void mystery1(int n) {
    for (int i = n; i > 0; i = i / 2) {
        for (int j = 0; j < 100000000; j += 2) {
            System.out.println("Hello World");
        }
    }
}
```

(b) Give the runtime of `mystery2(n)` in Θ notation.

```java
public void mystery2(int n) {
    for (int i = 1; i < n; i += 1) {
        for (int j = 0; j < n; j += 1) {
            i = i * 2;
            j = j * 2;
        }
    }
}
```

(c) Give the runtime of `mystery3(n)` in Θ notation.

```java
public void mystery3(int n) {
    for (int i = n; i > 0; i = i / 2) {
        for (int j = 1; j < i * i; j *= 2) {
            System.out.println("Hello World");
        }
    }
}
```

(d) Give the runtime of `mystery4(n)` in Θ notation. Assume that the `SLList` constructor, and the `size` and `addFirst` methods take constant time.

```java
public void mystery4(int n) {
    SLList<Integer> list = new SLList<>();
    for (int i = 1; list.size() < n; i += 1) {
        for (int j = 0; j < i; j += 1) {
            list.addFirst(j);
        }
        System.out.print(list.size() + " + ");
    }
}
```

3 WQU

(a) Draw the Weighted Quick Union object on 0 through 10, that results from the following `connect` calls. Do not use path compression. What is the resulting underlying array? Note that if we connect two sets of equal weight, by convention we make the set whose root has a smaller number the child of the other. We use the convention that if an element is the root of the set, its array value is the weight of the set negated.

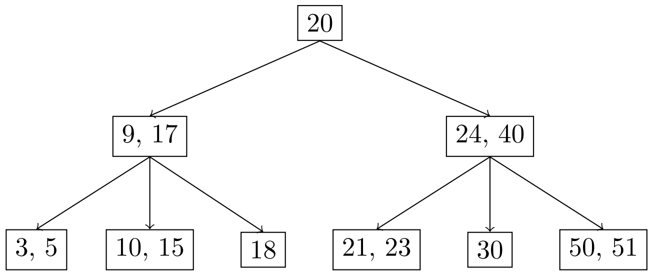
```
connect(0, 1);

connect(2, 3);

connect(9, 5);

connect(5, 7);

connect(7, 1);

connect(4, 2);

connect(3, 1);
```

(b) Assume that a single node has a height of 0. What are the shortest and tallest heights for a Quick Union object with 16 connected elements? What about for a Weighted Quick Union object?

(c) What are the best and worst runtimes for `connect` and `isConnected` in a Quick Union object with N connected elements? What about in a Weighted Quick Union object?

4 Switcheroo

(a) Consider the following 2-3 tree. Convert it to an LLRB, and describe the 6 LLRB operations to balance the tree after inserting the number 11. The LLRB operations are: `rotateRight(x)`, `rotateLeft(x)`, and `colorFlip(x)`.



(b) After inserting 11 and balancing the LLRB, how many red links are on along the longest path from the root to a leaf.

5 Mechanical Hashing

Suppose we insert the following words into an initially empty hash table, in this order: **kerfuffle**, **broom**, **hroom**, **ragamuffin**, **donkey**, **brekky**, **blob**, **zenzizenzizenzic**, and **yap**. Assume that the hash code of a String is just its length (note that this is not actually the hash code for Strings in Java). Use separate chaining to resolve collisions. Assume 4 is the initial size of the hash table's internal array, and double this array's size when the load factor is equal to 1. Illustrate this hash table with a box-and-pointer diagram.

For each index of the final hash table, specify what Strings are stored in it. If it is empty, write "none".